

記得寫上學號，班別及姓名等。請依題號順序每頁答一題。

1. (10%, 10%) The generalized dielectric constant.

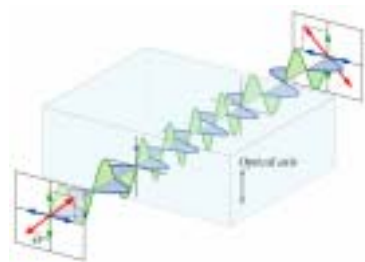
Assume there are  $N$  molecules per unit volume and  $Z$  electrons per molecule. Each molecule contains  $f_j$  electrons with binding frequency  $\omega_j$  and damping factor  $\gamma_j$ , where  $Z = \sum f_j$ . The electrons could be classified into two groups, bound electrons and free electrons. The free electrons are denoted as  $f_0$ ,  $\omega_0=0$ , and  $\gamma_0$ .

(a) Find the general polarization  $\mathbf{P}$ .

(b) Find the general form of the complex dielectric constant.

2. (10%, 10%) Birefringence: the wave plate

Linearly polarized light entering a wave plate can be resolved into two waves, parallel and perpendicular to the optical axis of the wave plate. In the plate, assume that the parallel wave ( $k_y$ ) propagates slightly slower than the perpendicular one ( $k_x$ ).



Near side:  $f_0 = A_0 \cos(k_x z_0 - \omega t) \hat{\mathbf{x}} + A_0 \cos(k_y z_0 - \omega t) \hat{\mathbf{y}}$

Far side:  $f_d = A_0 \cos(k_x(z_0 + d) - \omega t) \hat{\mathbf{x}} + A_0 \cos(k_y(z_0 + d) - \omega t) \hat{\mathbf{y}}$

(a) At the far side of the plate, can we change the polarization of the the resulting combination orthogonal to its entrance state? At what condition?

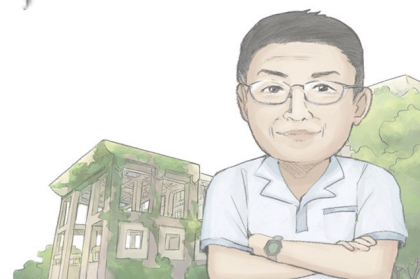
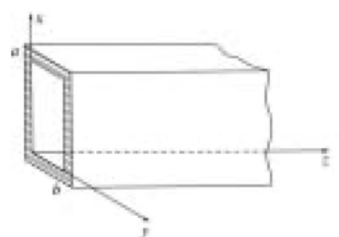
(b) Is it possible to form a right or left hand circular polarization? At what condition?

3. (10%, 10%) A wave is propagating in a rectangular waveguide with fundamental  $TE_{10}$  mode.

$$B_z(x, z, t) = B_0 \cos(\pi x / a) \cos(kz - \omega t).$$

(a) Find  $E_x$ ,  $E_y$ ,  $B_x$ , and  $B_y$ ?

(b) Find the surface current  $\mathbf{K}$  on the bottom of the inner wall (the  $yz$  plane)?



4. (10%, 10%) A coaxial transmission line of inner radius  $a$  and outer radius  $b$  is filled with a dielectric material of permittivity  $2\epsilon_0$  and permeability  $\mu_0$ .

(a) Find  $\mathbf{E}$  and  $\mathbf{B}$ .

(b) Find the electric  $u_e$  and magnetic  $u_m$  field density. Are they the same?

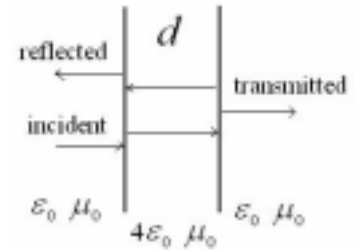
[Hint 1: Reduce the problem to two dimensions.]

[Hint 2:  $\nabla^2 \mathbf{E} = \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$ ,  $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ ,

$$\nabla \times \mathbf{v} = \left( \frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right) \hat{\mathbf{s}} + \left( \frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right) \hat{\boldsymbol{\phi}} + \frac{1}{s} \left( \frac{\partial}{\partial s} (s v_\phi) - \frac{\partial v_s}{\partial \phi} \right) \hat{\mathbf{z}}$$

5. (6%, 7%, 7%) A plane wave is normally incident onto a lossless dielectric slab of thickness  $d$  and dielectric constant  $4\epsilon_0$ , as shown in the figure.

Incident wave: 
$$\begin{cases} \mathbf{E}_I(z, t) = E_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \\ \mathbf{B}_I(z, t) = \frac{1}{v_1} E_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}} \end{cases}$$

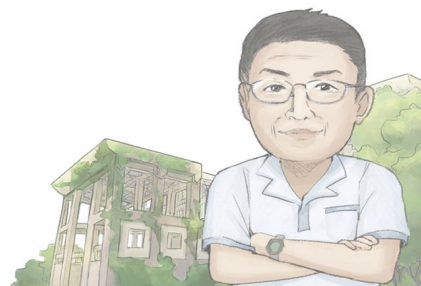


(a) Write down the reflected wave, transmitted wave, and wave in the dielectric slab.

(b) Determine the reflection coefficient  $R$  and the transmission coefficient  $T$  as a function of propagating constant  $k$  and thickness  $d$ .

(c) If the frequency of the incident wave is 30 GHz, what is the minimum thickness  $d$  such that no reflection is observed?

[Hint: Assume the media is linear and there is no free charge and no free current at the interface.]



(a)

Let  $\mathbf{x}(t) = \mathbf{x}_0 e^{-i\omega t}$  and substitute

$$\begin{cases} \mathbf{x}(t) = \mathbf{x}_0 e^{-i\omega t} \\ \mathbf{E}(\mathbf{x}, t) = \mathbf{E}(0) e^{-i\omega t} \end{cases} \text{ into } m(\ddot{\mathbf{x}} + \gamma \dot{\mathbf{x}} + \omega_0^2 \mathbf{x}) = -e\mathbf{E}(\mathbf{x}, t),$$

we obtain  $m(-\omega^2 - i\omega\gamma + \omega_0^2)\mathbf{x}_0 = -e\mathbf{E}(0)$  with the solution:

$$\mathbf{x}_0 = -\frac{e}{m} \frac{\mathbf{E}(0)}{\omega_0^2 - \omega^2 - i\omega\gamma} \Rightarrow \mathbf{x}(t) = -\frac{e}{m} \frac{\mathbf{E}(0) e^{-i\omega t}}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

Divide the electrons in the medium into  $\begin{cases} \text{bound electrons: } \omega_j \neq 0 \\ \text{free electrons: } \omega_j = 0, f_j = f_0, \gamma_j = \gamma_0 \end{cases}$

$$\mathbf{P}(\mathbf{x}) = N \sum_j f_j \mathbf{p}_j = \left( \frac{Ne^2}{m} \sum_{\text{bound}} \frac{f_j}{\omega_j^2 - \omega^2 - i\omega\gamma_j} + i \frac{Ne^2}{m} \frac{f_0}{-i\omega^2 + \omega\gamma_0} \right) \mathbf{E}(\mathbf{x}) = \varepsilon_0 \chi_e \mathbf{E}(\mathbf{x})$$

(b)

$$\begin{cases} \mathbf{D}(\mathbf{x}) \equiv \varepsilon_0 \mathbf{E}(\mathbf{x}) + \mathbf{P}(\mathbf{x}) = \varepsilon \mathbf{E}(\mathbf{x}) \\ \varepsilon = \varepsilon_0 (1 + \chi_e) \end{cases}$$

to fields with  $\exp(-i\omega t)$  dependence, we obtain  $\mathbf{D}(\mathbf{x}) = \varepsilon \mathbf{E}(\mathbf{x})$

$$\text{with } \varepsilon = \varepsilon_0 + \underbrace{\frac{Ne^2}{m} \sum_{j \text{ (bound)}} \frac{f_j}{\omega_j^2 - \omega^2 - i\omega\gamma_j}}_{\varepsilon_b} + i \underbrace{\frac{Ne^2 f_0}{m\omega(\gamma_0 - i\omega)}}_{\sigma/\omega} = \varepsilon_b + i \frac{\sigma}{\omega}$$

2.

(a) Yes, we can.

Near side:  $z_0=0$   $f_0 = A_0 \cos(-\omega t) \hat{\mathbf{x}} + A_0 \cos(-\omega t) \hat{\mathbf{y}}$ .

The field is polarized in the first and third quadrants.

Far side:  $f_d = A_0 \cos(k_y d + n\pi - \omega t) \hat{\mathbf{x}} + A_0 \cos(k_y d - \omega t) \hat{\mathbf{y}} = -A_0 \cos(k_y d - \omega t) \hat{\mathbf{x}} + A_0 \cos(k_y d - \omega t) \hat{\mathbf{y}}$ ,  
where  $(k_x - k_y)d = n\pi$ ,  $n = 1, 3, 5, \dots$

The resulting field is linearly polarized in the second and fourth quadrants.

(b) Yes, we can.

Near side:  $z_0=0$   $f_0 = A_0 \cos(-\omega t) \hat{\mathbf{x}} + A_0 \cos(-\omega t) \hat{\mathbf{y}}$ .

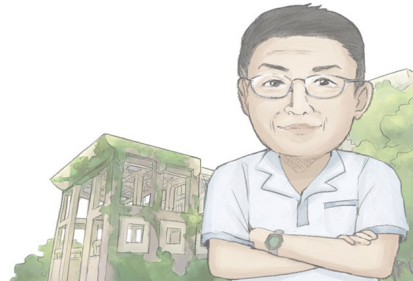
The field is polarized in the first and third quadrant.

Far side:  $f_d = A_0 \cos(k_y d + \frac{n}{2}\pi - \omega t) \hat{\mathbf{x}} + A_0 \cos(k_y d - \omega t) \hat{\mathbf{y}} = -A_0 \sin(k_y d - \omega t) \hat{\mathbf{x}} + A_0 \cos(k_y d - \omega t) \hat{\mathbf{y}}$ ,  
where  $(k_x - k_y)d = \frac{n}{2}\pi$ ,  $n = 1, 5, 9, \dots$  LHCP

Far side:  $f_d = A_0 \cos(k_y d + \frac{n}{2}\pi - \omega t) \hat{\mathbf{x}} + A_0 \cos(k_y d - \omega t) \hat{\mathbf{y}} = A_0 \sin(k_y d - \omega t) \hat{\mathbf{x}} + A_0 \cos(k_y d - \omega t) \hat{\mathbf{y}}$ ,  
where  $(k_x - k_y)d = \frac{n}{2}\pi$ ,  $n = 3, 7, 11, \dots$  RHCP

3.

$$\begin{aligned} E_x &= \frac{i}{(\omega/c)^2 - k^2} \left( k \frac{\partial E_z}{\partial x} + \omega \frac{\partial B_z}{\partial y} \right), & E_y &= \frac{i}{(\omega/c)^2 - k^2} \left( k \frac{\partial E_z}{\partial y} - \omega \frac{\partial B_z}{\partial x} \right) \\ B_x &= \frac{i}{(\omega/c)^2 - k^2} \left( k \frac{\partial B_z}{\partial x} - \frac{\omega}{c^2} \frac{\partial E_z}{\partial y} \right), & B_y &= \frac{i}{(\omega/c)^2 - k^2} \left( k \frac{\partial B_z}{\partial y} + \frac{\omega}{c^2} \frac{\partial E_z}{\partial x} \right) \end{aligned}$$



(a)  $B_z(x, z, t) = B_0 \cos(\pi x / a) \cos(kz - \omega t), E_z = 0$

$$E_x = \frac{i}{(\omega/c)^2 - k^2} (\omega \frac{\partial B_z}{\partial y}) = 0, \quad E_y = \frac{i}{(\omega/c)^2 - k^2} (-\omega \frac{\partial B_z}{\partial x}) = \frac{i}{(\omega/c)^2 - k^2} \frac{\omega \pi}{a} \sin(\frac{\pi}{a} x) \cos(kz - \omega t)$$

$$B_y = \frac{i}{(\omega/c)^2 - k^2} (k \frac{\partial B_z}{\partial y}) = 0, \quad B_x = \frac{i}{(\omega/c)^2 - k^2} (k \frac{\partial B_z}{\partial x}) = \frac{i}{(\omega/c)^2 - k^2} (-\frac{k \pi}{a}) \sin(\frac{\pi}{a} x) \cos(kz - \omega t)$$

(b) The surface current  $\mathbf{K}$  on the bottom of the inner wall.

At  $x=0$ , the only non-vanished term is  $B_z(x=0) = B_0 \cos(kz - \omega t)$ .

$$\mathbf{K} = \hat{\mathbf{n}} \times \mathbf{H} = \hat{\mathbf{x}} \times B_0 \cos(kz - \omega t) \hat{\mathbf{z}} = -B_0 \cos(kz - \omega t) \hat{\mathbf{y}}$$

4.

(a) The problem is reduced to two dimensions.

$$\left. \begin{aligned} \nabla_t \times \mathbf{E} &= 0 \\ \nabla_t \times \mathbf{B} &= 0 \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} \mathbf{E} &= -\nabla_t \phi_E, \quad \nabla_t^2 \phi_E = 0 \quad \text{electrostatic} \\ \mathbf{B} &= -\nabla_t \phi_B, \quad \nabla_t^2 \phi_B = 0 \quad \text{magnetostatic} \end{aligned} \right.$$

Electrostatic: the infinite line charge:  $\mathbf{E}_0(s, \phi, z, t) = \frac{A \cos(kz - \omega t)}{s} \hat{\mathbf{s}}$

Magnetostatic: an infinite straight current:  $\mathbf{B}(s, \phi, z, t) = \frac{B \cos(kz - \omega t)}{s} \hat{\phi}$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \rightarrow \left. \begin{aligned} \frac{\partial E_\phi}{\partial z} &= \frac{kA}{s} \sin(kz - \omega t) \\ -\frac{\partial B_s}{\partial t} &= \frac{\omega B}{s} \sin(kz - \omega t) \end{aligned} \right\} B = \frac{k}{\omega} A$$

(b)

$$u_e = \frac{1}{2} (2\epsilon_0) E^2 = \epsilon_0 \frac{A^2 \cos^2(kz - \omega t)}{s^2}$$

$$u_m = \frac{1}{2\mu_0} B^2 = \frac{1}{2\mu_0} \frac{B^2 \cos^2(kz - \omega t)}{s^2} = \epsilon_0 \frac{A^2 \cos^2(kz - \omega t)}{c^2 s^2} \left( \frac{1}{2\mu_0 \epsilon_0} \frac{\omega^2}{k^2} \right)$$

$$\text{since } \nabla^2 \mathbf{B} - \epsilon \mu \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \Rightarrow k^2 = 2\mu_0 \epsilon_0 \omega^2 \Rightarrow u_e = u_m$$

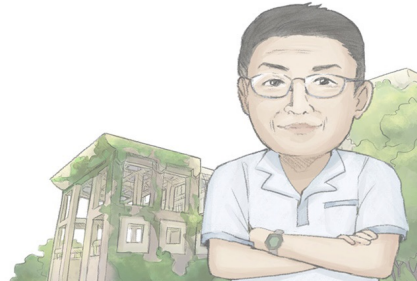
Yes.

5.

(a)

Incident wave:  $\left\{ \begin{aligned} \mathbf{E}_I(z, t) &= E_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \\ \mathbf{B}_I(z, t) &= \frac{1}{v_1} E_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}} \end{aligned} \right.$

Reflected wave:  $\left\{ \begin{aligned} \mathbf{E}_R(z, t) &= E_{0R} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \\ \mathbf{B}_R(z, t) &= -\frac{1}{v_1} E_{0R} e^{i(-k_1 z - \omega t)} \hat{\mathbf{y}} \end{aligned} \right.$



Wave in the dielectric slab: 
$$\begin{cases} \mathbf{E}_d(z, t) = E_{0c} e^{i(k_2 z - \omega t)} \hat{\mathbf{x}} + E_{0d} e^{i(-k_2 z - \omega t)} \hat{\mathbf{x}} \\ \mathbf{B}_d(z, t) = \frac{1}{v_2} E_{0c} e^{i(k_2 z - \omega t)} \hat{\mathbf{y}} - \frac{1}{v_2} E_{0d} e^{i(-k_2 z - \omega t)} \hat{\mathbf{y}} \end{cases}$$

Transmitted wave: 
$$\begin{cases} \mathbf{E}_T(z, t) = E_{0T} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \\ \mathbf{B}_T(z, t) = \frac{1}{v_1} E_{0T} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}} \end{cases}$$

(b)

Boundary condition:

Tangential E at  $z=0$ ,  $E_{0I} + E_{0R} = E_{0c} + E_{0d}$

Tangential B at  $z=0$ ,  $\frac{1}{v_1} (E_{0I} - E_{0R}) = \frac{1}{v_2} (E_{0c} - E_{0d})$

Tangential E at  $z=d$ ,  $E_{0c} e^{ik_2 d} + E_{0d} e^{-ik_2 d} = E_{0T} e^{ik_1 d}$

Tangential B at  $z=d$ ,  $\frac{1}{v_2} (E_{0c} e^{ik_2 d} - E_{0d} e^{-ik_2 d}) = \frac{1}{v_1} E_{0T} e^{ik_1 d}$

$$T = \left| \frac{E_{0T}}{E_{0I}} \right|^2 = 1 + \frac{1}{4} \left( \frac{k_1^2 - k_2^2}{k_1 k_2} \right)^2 \sin^2(k_2 d)$$

$$R = \left| \frac{E_{0R}}{E_{0I}} \right|^2 = \frac{1}{4} \left( 1 + \frac{1}{4} \left( \frac{k_1^2 - k_2^2}{k_1 k_2} \right)^2 \sin^2(k_2 d) \right) \left( \frac{k_2^2 - k_1^2}{k_1 k_2} \right)^2 \sin^2(k_2 d)$$

(c)

$$R = \frac{1}{4} \left( 1 + \frac{1}{4} \left( \frac{k_1^2 - k_2^2}{k_1 k_2} \right)^2 \sin^2(k_2 d) \right) \left( \frac{k_2^2 - k_1^2}{k_1 k_2} \right)^2 \sin^2(k_2 d) = 0$$

$$\sin(k_2 d) = 0 \Rightarrow k_2 d = \pi, \quad k_2 = \frac{4\pi f}{c}$$

$$d = \frac{c}{4f} = 0.25 \text{ cm}$$

